Supersymmetry Breaking[1]

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Abstract

These lectures provide a simple introduction to supersymmetry breaking. After presenting the basics of the subject and illustrating them in tree-level examples, we discuss dynamical supersymmetry breaking, emphasizing the role of holomorphy and symmetries in restricting dynamically-generated superpotentials. We then turn to mechanisms for generating the MSSM supersymmetry-breaking terms, including "gravity mediation", gauge mediation, and anomaly mediation. We clarify some confusions regarding the decoupling of heavy fields in general and D-terms in particular in models of anomaly-mediation.

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1 Introduction

Need we motivate lectures on supersymmetry breaking? Not really. If there is supersymmetry in Nature, it must be broken. But it's worth emphasizing that the breaking of supersymmetry, namely, the masses of superpartners, determines the way supersymmetry would manifest itself in experiment.

From a purely theoretical point of view, supersymmetry breaking is a very beautiful subject, and I hope these lectures will convey some of this beauty.

It is very hard to cover supersymmetry-breaking in three lectures. In the first lecture, section 2, we will describe the essentials of supersymmetry breaking. In the second lecture, section 3, we will study dynamical supersymmetry breaking. In the last lecture, section 4, we will describe several mechanisms for generating supersymmetry-breaking terms for the standard-model superpartners. This section can be read independently of section 3.

For lack of time, we will not cover supersymmetry-breaking mechanisms, or mechanisms for mediating the breaking, that rely on extra dimensions (we will discuss anomaly-mediation, because it is always present in four dimensions).

These lectures assume basic knowledge of supersymmetry (essentially the first seven chapters of Wess and Bagger[1], whose notations we will use). I tried to make section 3 self-contained, but a serious treatment of non-perturbative effects in supersymmetric gauge theories is beyond the scope of these lectures. For excellent reviews of the subject see, e.g., [2, 3, 4]. For more details and examples of dynamical supersymmetry breaking, see [5, 6]. Finally, ref. [7] is a comprehensive review of gauge-mediation models.

2 Basic features of supersymmetry breaking

In this section, we will discuss the fundamentals of supersymmetry breaking: the order parameters for the breaking, the Goldstone fermion, F-type and D-type tree-level breaking, and some general criteria for determining when supersymmetry is broken. The discussion will mostly be in the framework of $\mathcal{N}=1$ global supersymmetry, but we will end this section by commenting on how things are modified for local supersymmetry.

2.1 Order parameters for supersymmetry breaking

When looking for spontaneous supersymmetry breaking, we are asking whether the variation of some field under the supersymmetry transformations is non-zero in the ground state,

$$\langle 0|\delta(\text{field})|0\rangle \neq 0 \ . \tag{1}$$

For a chiral superfield ϕ, with scalar component $\tilde{\phi}$, fermion component ψ, and auxiliary component F, the supersymmetry variation are roughly (omitting numerical coefficients),

$$\delta_\xi \tilde{\phi}(x) \quad \sim \quad \xi\psi(x)$$

$$
\begin{aligned}
\delta_\xi \psi(x) &\sim i\sigma^\mu \bar{\xi}\, \partial_\mu \tilde{\phi}(x) + \xi F(x) \\
\delta_\xi F(x) &\sim i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi(x) \ ,
\end{aligned}
\tag{2}
$$

where ξ parameterizes the supersymmetry variation. Clearly, the only Lorentz invariant on the RHS of eqn. (2) is F, so supersymmetry is broken if

$$
< F > \neq 0 \ , \tag{3}
$$

and the field whose variation is non-zero in this case is the fermion, $\langle 0|\delta_\xi \psi(x)|0\rangle \neq 0$.

Similarly, for the vector superfield, only the gaugino variation can be non-zero

$$
\langle 0|\delta_\xi \lambda(x)|0\rangle \propto \langle 0|D|0\rangle \neq 0 \ , \tag{4}
$$

so a non-zero $\langle D\rangle$ signals supersymmetry breaking.

A much more physical order parameter for global supersymmetry breaking is the vacuum energy. The supersymmetry algebra contains the translation operator P_μ

$$
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}}\, P_\mu \ , \tag{5}
$$

where Q is the supersymmetry generator. Therefore the Hamiltonian H can be written as

$$
H = \frac{1}{4}\,(\bar{Q}_1 Q_1 + \bar{Q}_2 Q_2 + \text{h.c.}) \ . \tag{6}
$$

Since this is a positive operator, the energy of a supersymmetric system is either positive or zero. Furthermore, if supersymmetry is unbroken, the vacuum is annihilated by the supersymmetry generators, and

$$
E_{\text{vacuum}} = \langle 0|H|0\rangle = 0 \ . \tag{7}
$$

Thus, a non-zero vacuum energy signals spontaneous supersymmetry breaking.

In order to know whether global supersymmetry is spontaneously broken, we therefore need to study the minima of the scalar potential, and see whether there is a minimum with zero energy.

2.2 The scalar potential and flat directions

In a theory with chiral superfields ϕ_i, superpotential $W(\phi_i)$ and Kähler potential $K(\phi_i, \phi_i^\dagger)$, the scalar potential is given by

$$
V_F = K^{-1}_{i*j} \frac{\partial W^*}{\partial \phi_i^*} \frac{\partial W}{\partial \phi_j} = K^{-1}_{ij} F_i^* F_j \ , \tag{8}
$$

where

$$
K_{ij*} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_j *} \ . \tag{9}
$$

In eqn. (8) we used the fact that, on-shell, the auxiliary fields are given by

$$
F_i = \frac{\partial W}{\partial \phi_i} \ . \tag{10}
$$

If there are gauge interactions in the theory the scalar potential has additional contributions and is given by

$$V = V_F + V_D = V_F + \frac{1}{2} g^2 \sum_a (D^a)^2 \ , \tag{11}$$

where $D^a = \sum_i \phi_i^\dagger T^a \phi_i$. As expected, the scalar potential is non-negative, and again we see that supersymmetry is broken by a non-zero F and /or D vacuum expectation value (VEV). Only then is the ground state energy non-zero.

To look for the zeros of the scalar potential (in field space) in a theory with gauge interactions, we need to do the following:

1. Find the sub-(field)space for which $D^a = 0$. This is often called the space of "D-flat directions". Note that along these directions, the potential is not merely flat, but rather zero[2]. The space of D-flat directions can be parametrized by the VEVs of the chiral gauge invariants that one can construct from the fundamental chiral fields of the theory. This is an extremely useful result and we will often use it in the following.

2. If for a subspace of the D-flat directions we also have $F_i = 0$ (for all F_i's), then the potential is zero. The sub-(field) space for which this happens is often called the "moduli space".

To look for supersymmetry breaking, we will be interested then in the moduli-space of the theory. If there is no moduli space, supersymmetry is broken.

Exercise: D-flat directions: Consider an $SU(N)$ gauge theory with chiral fields $Q_i \sim N$, $\bar{Q}^A \sim \bar{N}$, with $i, A = 1, \ldots, F$. (This theory is usually called $SU(N)$ with F flavors.) Assume $F < N$. Denote the $SU(N)$ gauge index by α. Show that

$$Q_{i\alpha} = \bar{Q}_{i\alpha} = v_i \delta_{i\alpha} \ , \tag{12}$$

are D-flat. The D-flat directions of the theory are then given by (12) up to global $SU(F)_L \times SU(F)_R$ and gauge rotations.

As mentioned above, an alternative parameterization of the D-flat directions is in terms of the VEVs of the gauge invariants of the theory. In this case, the only chiral gauge invariants are the "mesons" $M_i^A = Q_i \cdot Q^A$. Indeed, using the global symmetry we can always write the meson VEVs as

$$M_i^A = \mathrm{diag}(V_1, V_2, \ldots, V_N) \ . \tag{13}$$

and the two parameterizations are clearly equivalent $V_i \leftrightarrow v_i^2$.

[2] The reason why these directions are called "flat" will become clear once we discuss radiative corrections. Typically, in non-supersymmetric theories, if we have a flat potential at tree level, the degeneracy is lifted by radiative corrections. As we will see, in supersymmetric theories, if the ground state energy is zero at tree-level, it remains zero to all orders in perturbation theory. Therefore, the directions in field space for which $V = 0$ are the only ones that are truly flat—they remain zero to all orders in perturbation theory.

2.3 The Goldstino

With broken supersymmetry $Q_\alpha|0\rangle$ is non-zero. What is it then? The generator of a broken bosonic global symmetry gives the Goldstone boson. Likewise, $Q_\alpha|0\rangle$ gives the Goldstone fermion of supersymmetry breaking, or "Goldstino", which we denote by $\psi^G_\alpha(x)$.

To see the Goldstino concretely, we should examine the supersymmetry current, and look for a piece that is linear in the fields. The supersymmetry current is of the form

$$J^\mu_\alpha \sim \sum_\phi \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}\,(\delta\phi)_\alpha \ , \tag{14}$$

where $\delta\phi$ is the supersymmetry variation of the the field ϕ. Since $\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}$ cannot get a VEV, a term that is linear in the fields can only occur when $\delta\phi$ gets a non-zero VEV. As we saw before, the only fields whose supersymmetry variations can have non-zero VEVS are the fermion of the chiral superfield, ψ (the VEV of whose variation is F), and the fermion of the vector superfield, λ (the VEV of whose variation is D). Thus,

$$J^\alpha_\mu \sim \sum_i \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi_{i\alpha})}\,\langle F_i\rangle + \frac{1}{\sqrt{2}}\sum_a \frac{\delta\mathcal{L}}{\delta(\partial_\mu\lambda^a_\alpha)}\,\langle D^a\rangle \ , \tag{15}$$

so that

$$\psi^G_\mu \sim \sum_i \langle F_i\rangle\psi_i + \sum_a \langle D^a\rangle\lambda^a \ . \tag{16}$$

We see that the Goldstino is a combination of the fermions that correspond to non-zero auxiliary field VEVs.

To demonstrate the basics we have seen so far, let us now turn to two examples of supersymmetry breaking. These examples will also illustrate some other general features of supersymmetry breaking.

2.4 Tree-level breaking: F-type

In this section we will study a variation of the O'Raifeartaigh model [8], with chiral fields Y_i, Z_i, and X with $i = 1, 2$, with the superpotential

$$W = X(Y_1Y_2 - M^2) + m_1Z_1Y_1 + m_2Z_2Y_2 \ , \tag{17}$$

where M and m_i are parameters with the dimension of mass. Note that the superpotential has a term that is linear in one of the fields (X). This is crucial for breaking supersymmetry at tree-level.

The original O'Raifeartaigh model is obtained by identifying $Y_1 = Y_2 = Y$, and $X_1 = X_2 = X$. We are complicating the model in order to illustrate the interplay between broken global symmetries and supersymmetry breaking, which we will get to later. But let's postpone that, and see whether the model breaks supersymmetry.

Since there are no gauge interactions in the model, we don't have to worry about D-terms, and we can turn directly to finding whether there are F-flat directions for which the potential vanishes. Equating all the F-terms to zero we have the following equations:

$$\begin{array}{lll} 1 & Y_1Y_2 = M^2 & (F_X) \\ 2 & XY_2 + m_1Z_1 = 0 & (F_{Y_1}) \\ 3 & XY_1 + m_2Z_2 = 0 & (F_{Y_2}) \\ 4 & m_1Y_1 = 0 & (F_{Z_1}) \\ 5 & m_2Y_2 = 0 & (F_{Z_2}) \end{array}$$

Clearly, equations 4 and 5 clash with equation 1. There is no point for which the potential vanishes, and supersymmetry is broken. Note that it is crucial that M, m_1 and m_2 are all non-zero. If $M = 0$, there is no linear term in the superpotential, and the origin of field space is always a supersymmetric point[3]. If for example, $m_2 = 0$, we can have a solution with $Y_1 \to 0$ and $Y_2 \to \infty$, such that their product is M^2.

You may be gasping with disbelief at how simple supersymmetry breaking is. And it's true: given a superpotential, finding out whether supersymmetry is broken simply amounts to solving a system of equations. The tricky part, as we will see, is to derive the superpotential, which usually involves understanding the dynamics of the theory.

As we saw above, supersymmetry is broken in this model. Very often, this is all one can say about a model. There are many other questions one can ask, such as: Where is the minimum of the potential? Which global symmetries are preserved in this minimum? What is the ground state energy? What is the light spectrum? To answer these questions, we need to know the Kähler potential of the theory.

In fact, we have already made an implicit assumption about the Kähler potential when we determined that supersymmetry is broken. We found that some F terms are non-zero in the model, but inspecting (8), we see that the potential can still vanish if K_{ij} blows up. So we are assuming that the Kähler potential is well behaved. For the simple chiral model we wrote above, this is a completely innocent assumption. But in general, when we study gauge theories with complicated dynamics, this is an important caveat to keep in mind.

But let's take the tree-level Kähler potential of our toy model to be canonical. The potential is then

$$V = \left|Y_1Y_2 - M^2\right|^2 + \left[\left|XY_2 + m_1Z_1\right|^2 + m_1^2\left|Y_1^2\right|^2 + 1 \leftrightarrow 2\right] \ . \tag{18}$$

Exercise: Show that for $m_, m_2 \ll M$, the potential is minimized along

$$\langle Y_1 \rangle \quad = \quad v_1 \equiv \sqrt{\frac{m_2}{m_1}}\sqrt{M^2 - m_1m_2}$$

[3] This will no longer hold when we discuss non-perturbative effects, which can give superpotential terms with negative powers of the fields.

$$Z_1 = -\frac{1}{m_1} X Y_2 \ , \qquad (19)$$

and similarly for $1 \leftrightarrow 2$.

Instead of an isolated minimum, there is a direction in field space for which V is constant and non-zero. This is typical of O'Raifeartaigh like models. The degeneracy is removed at the loop level. For example, in our toy model the true minimum will occur at $X = Z_i = 0$.

We now turn to a useful criterion for supersymmetry breaking [10, 11]. Suppose a theory has

1. A spontaneously broken global symmetry
2. No classical flat directions

then supersymmetry is broken.

Let us illustrate this in our toy model. As we saw above, the model has no flat directions. Furthermore, there is a $U(1)$ global symmetry, under which we can choose the charges to be

$$X(0) \ \ Y_1(1) \ \ Y_2(-1) \ \ Z_1(-1) \ \ Z_2(1) \ . \qquad (20)$$

Take for simplicity $m_1 = m_2 = m << M$. The ground state is at

$$\langle Y_1 \rangle = \langle Y_2 \rangle = v = \sqrt{M^2 - m^2} \ . \qquad (21)$$

So the $U(1)$ is broken and there is a massless Goldstone boson, which we can parameterize as ϕ_R with

$$\begin{aligned} Y_1 &= v e^{i(\phi_R + i\phi_I)} \\ Y_2 &= v e^{-i(\phi_R + i\phi_I)} \ . \end{aligned} \qquad (22)$$

Consider the potential at $X = Z_i = 0$,

$$V = \left| Y_1 Y_2 - M^2 \right|^2 + m^2 \left(|Y_1|^2 + |Y_2|^2 \right) \ . \qquad (23)$$

As expected, ϕ_R drops out, but ϕ_I doesn't. However, for the supersymmetric theory with $m = 0$, ϕ_I drops out too. What we are seeing of course is that the supersymmetric theory is invariant under the "complexified" $U(1)$. With unbroken supersymmetry, the massless Goldstone boson is part of a massless chiral superfield, so there must be an *additional* massless real scalar, and together they form a complex scalar. In our example, the Goldstone is ϕ_R, and it corresponds to a compact flat direction. In the supersymmetric theory ($m = 0$), the Goldstone is accompanied by another massless scalar, ϕ_I, which corresponds to a non-compact flat direction. When $m \neq 0$, there is no non-compact flat direction and therefore no other massless scalar. Thus, the Goldstone cannot be part of a supersymmetric multiplet and supersymmetry must be broken.

In our toy example, it was easy to verify directly that supersymmetry is broken. But in some examples, where a direct analysis is impossible, it is still possible to show that there are no classical flat directions, and that the global symmetry of the model is broken, and thus to conclude that supersymmetry is broken. We will see such an example in the next lecture.

2.5 Tree-level breaking: D-type

In this section we will study the Fayet-Iliopulos model [9], in which supersymmetry is broken by a non-zero D term (and/or F term). The model has a $U(1)$ gauge symmetry. The important observation is that the auxiliary field of the $U(1)$ vector field is gauge invariant, and therefore can appear in the Lagrangian. (From the point of view of supersymmetry, we can always add an auxiliary field to the Lagrangian because its supersymmetry variation is a total derivative.) Consider then a model with chiral superfields Q and $\bar{Q}$, whose $U(1)$ charges are 1 and -1 respectively, and with the Kähler potential

$$K = Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} \bar{Q} + \xi_{\rm FI} V \ , \tag{24}$$

and superpotential

$$W = mQ\bar{Q} \ , \tag{25}$$

where V is the vector superfield.

The potential is

$$V = \frac{1}{2} g^2 \left[|Q|^2 - |\bar{Q}|^2 + \xi_{\rm FI} \right]^2 + m^2 \left[|Q|^2 + |\bar{Q}|^2 \right] \ . \tag{26}$$

We see that the potential is never zero. For the D-part to vanish we need $\langle \bar{Q} \rangle \neq 0$, but then the F_Q term

$$\frac{\partial W}{\partial Q} = m\bar{Q} \neq 0 \ . \tag{27}$$

Exercise: Show that the minimum is at

1. $\langle Q \rangle = \langle \bar{Q} \rangle = 0$ for $g^2 \xi_{\rm FI} < m^2$. In this case the $U(1)$ is unbroken, the D-term is non-zero, but all F-terms vanish.

2. $\langle Q \rangle = 0$, $\langle \bar{Q} \rangle = v = \sqrt{2}\sqrt{\xi_{\rm FI} - m^2/g^2}$ for $g^2 \xi_{\rm FI}^2 > m^2$. In this case the $U(1)$ is broken, the D-term is non-zero, and one F-term is non-zero.

Exercise: Show that the Goldstino is

$$\psi_G \sim m\lambda + \frac{i}{2} g v \psi_Q \ , \tag{28}$$

where λ is the gaugino and ψ_Q is the Q-fermion. We explicitly see that the Goldstino is a combination of fermion fields whose F- or D-terms are non-zero.

2.6 Going local

So far we only discussed global supersymmetry, so let us briefly mention which parts of our discussion above are modified when we promote supersymmetry to a local symmetry. For lack of time and space, we just present here the results. Although we can't see the origin of these results, they are still useful in order to

understand, at least qualitatively, what Nature looks like if it has spontaneously broken supersymmetry.

• The order parameter for F-type breaking now becomes

$$D_\phi W = \frac{\partial W}{\partial \phi} + \frac{1}{M_{Pl}^2}\frac{\partial K}{\partial \phi} W \ . \tag{29}$$

If we decouple gravity by taking the Planck scale M_{Pl} to infinity, this reduces to (3).

• The vacuum energy is no-longer an order parameter for supersymmetry breaking. This is very fortunate, because we certainly don't want the cosmological constant to be of the order of the supersymmetry breaking scale. The scalar potential is now (omitting D-terms)

$$V = e^{K/M_{Pl}^2}\left[(D_i W)^* K_{ij}^{-1}(D_j W) - \frac{3}{M_{Pl}^2}|W|^2\right] \ . \tag{30}$$

We can always shift the superpotential by a constant, $W(\phi) \to W(\phi) + W_0$ so that $V = 0$ even when $D_i W = 0$.

• When supersymmetry is broken, the gravitino gets a mass. The Goldstino is eaten by the gravitino, and supplies the extra two degrees of freedom required for a massive gravitino.

• The supergravity multiplet contains the graviton, gravitino, and auxiliary fields. When supersymmetry is broken, the scalar auxiliary field of the supergravity multiplet acquires a VEV.

In the last lecture, when we discuss how supersymmetry breaking terms are generated for the minimal supersymmetric standard model (MSSM), we will need to know how a non-zero VEV of the supergravity scalar auxiliary field affects the MSSM fields. So we need to know how this auxiliary field couples to chiral and vector fields. It is convenient to parameterize this auxiliary field as the F-component of a *non-dynamical* chiral superfield[4]

$$\Phi = 1 + F_\Phi \theta^2 \ . \tag{31}$$

The supergravity auxiliary field then couples to chiral and vector superfields through the following rescaling of the usual Lagrangian.

$$\mathcal{L} = \int d^\theta \Phi^3 W(Q) + \int d^4\theta \Phi^\dagger \Phi K(Q^\dagger, e^V Q) + \int d^2\theta \tau W^\alpha W_\alpha \ . \tag{32}$$

It is easy to see from this that Φ is related to scale transformations. We can also see from the Lagrangian (32) that when supersymmetry is broken, F_Φ becomes non-zero. Equation (32) will be our starting point when we discuss anomaly mediated supersymmetry breaking in section 4.2.

[4]This field is called the chiral compensator, because it is often introduced in order to write down a superspace Lagrangian for supergravity that is manifestly invariant under Weyl-rescaling. Note that the lowest component of Φ breaks the "fake" Weyl invariance. Non-dynamical fields of this type, which are introduced in order to make the Lagrangian look invariant under some fake symmetry, are called spurions.

3 Beyond tree level: dynamical supersymmetry breaking

Consider a supersymmetric gauge theory with some tree-level superpotential W_{tree}, and with a minimum at zero energy, $V_{\text{tree}} = 0$. Then the ground state energy remains zero to all orders in perturbation theory [14, 15, 16]. This follows from the "non-renormalization" of the superpotential— the tree-level superpotential is not corrected in perturbation theory, which in turn, follows from the fact that the superpotential is a holomorphic function of the fields [17]. We will not prove here this non-renormalization theorem, but we will see in detail two examples of how holomorphy and global symmetries dictate the form of the superpotential in section 3.2. It will be clear in these examples that the tree-level superpotential is not corrected radiatively.

This leads to one of the most important results about supersymmetry breaking: If supersymmetry is unbroken at tree-level, it can only be broken by non-perturbative effects. Only the dynamics of the theory can generate a non-zero potential. This makes the study of supersymmetry breaking hard (and interesting!). However, holomorphy, which forces us to consider non-perturbative phenomena when studying supersymmetry breaking, also comes to our aid. As we will see, we can say a lot about the dynamics of supersymmetric theories based on holomorphy.

Before going on, let us pause to say a word about one kind of non-perturbative phenomenon—instantons[5], which we will encounter in the following. Instantons are classical solutions of the Euclidean Yang-Mills action that approach pure gauge for $|x| \to \infty$. Therefore, the field strength for these solutions goes to zero at infinity, and the instanton action is finite. The one-instanton action is

$$S_{\text{inst}} = \frac{1}{2g^2} \int d^4x F_{\mu\nu}^2 \sim \frac{8\pi^2}{g^2} \ , \tag{33}$$

where g is the gauge coupling. If there are fermions charged under the gauge group, instantons can generate a fermion interaction with strength proportional to the instanton action, $\exp(-8\pi^2/g^2)$. The gauge coupling is of course scale-dependent, and obeys at one-loop

$$\mu \frac{dg}{d\mu} = -\frac{b}{16\pi^2}\, g^3 \tag{34}$$

(In our conventions, $\mathcal{N} = 1$ $SU(N)$ with F flavors has $b = 3N - F$.) So the instanton-generated interactions involve

$$\exp\left(-\frac{8\pi^2}{g^2(\mu)}\right) = \frac{\Lambda^b}{\mu^b} \ , \tag{35}$$

where Λ is the strong coupling scale of the theory.

[5] This is intended for students who have never heard about instantons, and would still like to follow these lectures. It is by no means a serious introduction to instantons, and I refer you to [4] for an introduction to instantons in supersymmetric gauge theories.

In an $SU(N)$ theory with $F = N - 1$ flavors, instantons generate fermion-scalar interactions that can be encoded by the superpotential [12]

$$W_{np} = \left(\frac{\Lambda^{3N-F}}{\det(Q \cdot \bar{Q})} \right)^{\frac{1}{N-F}} \quad . \tag{36}$$

We will study this example in detail below.

Going back to supersymmetry breaking, we see that if the ground state energy is zero at tree-level (unbroken supersymmetry), only dynamical effects can alter that, and therefore the full ground state energy, or supersymmetry-breaking scale, is proportional to some strong coupling scale Λ. This has a profound implication: If a theory breaks supersymmetry spontaneously, with supersymmetry unbroken at tree-level, then the supersymmetry breaking scale, or the ground state energy, is proportional to some strong coupling scale Λ,

$$E_{\rm vac} \sim \Lambda \sim M_{UV}\, e^{-\frac{8\pi^2}{g^2(M_{UV})}} \ , \tag{37}$$

where M_{UV} is the cutoff scale of the theory, say, M_{Pl}. Thus, supersymmetry can do much more than *stabilize* the Planck-electroweak scale hierarchy. It can actually *generate* this hierarchy if it's broken dynamically [13], because the factor $e^{-\frac{8\pi^2}{g^2(M_{UV})}}$ can easily be 10^{-17}.

In general, there are three types of (dynamical) supersymmetry-breaking models.

1. In some models we can only tell that supersymmetry is broken based on indirect arguments. In particular, we have no information about the potential of the theory, and all we know is that the supersymmetry-breaking scale is of the order of the relevant strong-coupling scale.

2. In some models, we can derive the superpotential at low-energies (in variables such that the Kähler potential is non-singular), and conclude that some F-terms are non-zero. Such models are often called "non-calculable", because apart from determining that supersymmetry is broken, we cannot calculate any of the properties of the ground state (including the supersymmetry breaking scale).

   How do we determine the superpotential in these models? There are many methods, some of which we will see today. These typically involve holomorphy, global symmetries, known exact results and even Seiberg duality.

3. In some models, we can calculate the superpotential as above, but for certain ranges of parameters, the theory is weakly coupled and we can also calculate the Kähler potential. Then we can compute the supersymmetry breaking scale, the light spectrum, and other properties of the ground state.

   Roughly, these models have the following behavior. There is a tree-level superpotential W_{tree}, with some couplings λ, that lifts all flat directions

(classically). Because W_{tree} is a polynomial in the fields, it vanishes at the origin, and grows for large field VEVs. On the other hand, non-perturbative effects generate a potential that is strong in the origin of field space, but decreases for large field VEVs (because the gauge symmetry is Higgsed with large scalar VEVs, so the low energy is weakly coupled). The interplay between the tree-level potential and the non-perturbatively generated potential may give a supersymmetry breaking ground state. Clearly, if we decrease the tree-level coupling λ, V_{tree} becomes smaller, so that the ground state is obtained at larger values of the field VEVs, where the theory is weakly coupled.

In the remainder of this section, we will demonstrate this through two examples out of the many known supersymmetry breaking models. We will spend most of our time studying the $3-2$ model. This example will illustrate how holomorphy, symmetries and known results about the superpotentials of various theories, completely determine the superpotential of the model.

3.1 Indirect analysis—$SU(5)$ with single antisymmetric

We will now see an example of the first type of models discussed above, where there is only indirect evidence for supersymmetry breaking. We will apply here the criterion explained in section 2.4: If a theory has broken global symmetries and no flat directions, supersymmetry is broken. Our example is an $SU(5)$ gauge theory with fields $T \sim 10$, $\bar{F} \sim \bar{5}$ [10, 18]. As explained above, the D-flat directions of a gauge theory can be parametrized by the chiral gauge invariants. Since we cannot form any gauge invariants out of T and $\bar{F}$, there are no flat directions.

The global anomaly-free symmetry of the model is $G = U(1) \times U(1)_R$, with charges $T(1,1)$ and $\bar{F}(-3,-9)$. We can now argue, based on 't Hooft anomaly matching, that G is spontaneously broken.

So let's show that G is (most likely) broken. First, the $SU(5)$ theory probably confines. (We stress that we cannot prove this, but since this $SU(5)$ is asymptotically free, with few matter fields, this is a very likely possibility.) Suppose then that the global symmetry is unbroken. Then the $SU(5)$-invariant composite fields of the confined theory should reproduce the global anomalies, $U(1)^3$, $U(1)^2U(1)_R$, etc of the original theory. Denoting the fields of the confined theory by X_i, and their charges under G by (q_i, r_i), we obtain four equations for the q_i's and r_i's. There is no simple solution to these equations. Allowing only charges below 50, we need at least 5 fields to obtain a solution. We conclude then that the global symmetry is (probably) broken. Since there are no classical flat directions, supersymmetry is (probably) broken.

The supersymmetry breaking scale is proportional to the only scale in the problem, which is the strong coupling scale of $SU(5)$.

3.2 Direct analysis: the $3-2$ model

The $3-2$ model is probably the canonical example of supersymmetry breaking [11]. It is certainly one of the simplest models in the sense that it has a small gauge group $SU(3) \times SU(2)$, and relatively small field content. But it is actually not the simplest model to analyze. Still, this makes it an interesting example, and we will use it to demonstrate several important points. We will see how the superpotential is determined by holomorphy and symmetries. The basic observation we will use is that the parameters of the theory can be thought of the VEVs of background fields. The notion of holomorphy can then be extended to these parameters.

Furthermore, this model will also demonstrate the three types of analysis detailed in the beginning of this section. We will first establish supersymmetry breaking by the indirect argument we saw in section 2.4: we will show that the model has no flat directions and a broken global symmetry. We will then derive the exact superpotential of the theory and show that it gives at least one non-zero F-term. Finally, we will choose parameters such that the minimum is calculable.

3.2.1 Classical theory

The field content of the model is $Q \sim (3,2)$, $\bar{Q}_A \sim (\bar{3},1)$, $L \sim (1,2)$ with $A=1,2$. We add the superpotential

$$W_{\rm tree} = \lambda Q \cdot \bar{Q}_2 \cdot L \ . \tag{38}$$

As explained in section 2.2, we should first find the D-flat directions, and these can be parametrized by the classical gauge-invariants that we can make out of the chiral fields

$$\begin{aligned} X_A &= Q \cdot \bar{Q}_A \cdot L = Q_{i\alpha}\bar{Q}^i_A L_\beta \epsilon^{\alpha\beta} & (39) \\ Y = \det(Q \cdot \bar{Q}) &= \epsilon^{\alpha\beta}\epsilon^{AB}\,(Q_{i\alpha}\bar{Q}^i_A)\,Q_{j\beta}\bar{Q}^j_B \ , & (40) \end{aligned}$$

where i (α) is the $SU(3)$ ($SU(2)$) gauge index. To see this, it is easy to start by making $SU(3)$ invariants: $Q_\alpha \cdot \bar{Q}_A$. These are $SU(2)$ doublets, and together with the remaining doublet L_α, they can be combined into the $SU(2)$ invariants X_A and Y.

Next, we should find the subspace of the D-flat directions for which all F-terms vanish. Consider for example the requirement that the L F-term vanishes,

$$\frac{\partial W}{\partial L_\alpha} = \lambda Q_\alpha \cdot \bar{Q}_2 = 0 \ . \tag{41}$$

Contracting this equation with L_α we see that $X_2 = 0$. Similarly, you can show that $X_1 = Y = 0$. Thus, there are no flat directions classically–only the origin is a supersymmetric point.

Remembering our indirect criterion of section 2.4, let's consider the global symmetry of the model. The only anomaly-free symmetry that's preserved by

the superpotential (38), is $U(1)\times U(1)_R$, with charges $Q(1/3,1)$, $\bar{Q}_1(-4/3,-8)$, $\bar{Q}_2(2/3,4)$, and $L(-1,-3)$. If we can show that this global symmetry is broken, we'll know that supersymmetry is broken.

3.2.2 Exact superpotential

So let's turn to the quantum theory. We already know that only non-perturbative effects can change the potential (and in particular "lift" the classical zero potential at the origin). We also mentioned that the tricky part is to find the proper variables, for which the Kähler potential is well behaved. Our first task is then to find such variables and derive the superpotential [19]. Let's first see if we missed any gauge invariants. The way we constructed the gauge invariants above was to contract $SU(3)$ indices first. What happens if we do it the other way around? We find one new gauge invariant

$$Z = (Q^2)\cdot(Q\cdot L) = \epsilon^{ijk}Q_{i\alpha}Q_{j\beta}\epsilon^{\alpha\beta}Q_{k\gamma}L_\delta\epsilon^{\gamma\delta} \ . \tag{42}$$

Note that $Z=0$ classically.

We turn now to deriving the superpotential. Beyond tree level, there can be contributions to the superpotential generated by the $SU(3)$ and $SU(2)$ dynamics. To analyze these, it is useful to consider various limits.

Take first $\Lambda_3 >> \Lambda_2$, and λ much smaller than the gauge couplings. Then we have an $SU(3)$ theory with two flavors. An $SU(3)$ instanton then gives rise to the superpotential

$$W_3 = \frac{\Lambda_3^7}{Y} \ . \tag{43}$$

Below we will see that (43) is the most general superpotential allowed by the symmetries of the theory.

But before doing that, let's note that we can already conclude that supersymmetry is broken! As a result of the the superpotential (43), the ground state is at non-zero Y. But Y appears in the superpotential, so its R-charge must be non-zero (you can check that it is indeed -2). Therefore the global R-symmetry is broken, and since there are no flat directions, supersymmetry must be broken too.

Note the difference between an R- and non-R symmetry in this respect. We were able to conclude that the R symmetry is broken because a certain superpotential term is non-zero at the ground state, and any superpotential term is charged under the R-symmetry (assuming of course that there is an R symmetry that the superpotential preserves). Since the superpotential is neutral under non-R symmetries, we cannot conclude analogously that a non-R symmetry is broken.

Let us now show that the $SU(3)$ superpotential must be of the form (43). In fact, we will show this more generally for an $SU(N)$ gauge theory with $F<N$ flavors Q and $\bar{Q}$. The global symmetry of this theory is $SU(F)_L\times SU(F)_R\times U(1)_B\times U(1)_R$, with $Q\sim(F,1,1,(F-N)/F)$, and $\bar{Q}\sim(1,\bar{F},-1,(F-N)/F)$. The superpotential must be gauge invariant, so it can only depend on the

"mesons", $M_{ij} = Q_i \cdot \bar{Q}_j$ (with a slight abuse of notation, we are using now Latin indices to denote both $SU(F)_L$ and $SU(F)_R$ indices, with $i, j = 1, \ldots, F$). So $W = W(M_{ij})$.

Furthermore, the superpotential better be invariant under $SU(F)_L \times SU(F)_R$, so $W = W(\det M)$, where M stands for the meson matrix. Now $\det M$ is neutral under $U(1)_B$, but has $U(1)_R$ charge $2(F-N)$. Therefore

$$W \propto \left(\frac{1}{\det \bar{Q} \cdot Q}\right)^{\frac{1}{N-F}} \quad . \tag{44}$$

The only other thing W can depend on is the $SU(N)$ scale Λ^{3N-f}, so on dimensional grounds it is of the form

$$W = \text{const} \left(\frac{\Lambda^{3N-F}}{\det \bar{Q} \cdot Q}\right)^{\frac{1}{N-F}} \quad . \tag{45}$$

Note that holomorphy was crucial in this argument—without it we could make invariants such as $Q^\dagger Q$. Also note that we have just proven the non-renormalization theorem for this theory. We did not put in any tree-level superpotential, so $W_{tree} = 0$. We argued that (45) is the most general form of the superpotential in the quantum theory. But radiative corrections can only produce positive powers of the fields. So indeed the tree-level superpotential is not corrected radiatively.

Of course, we have only shown that the superpotential (45) is allowed. We haven't shown that it is actually generated, because that's much harder [12, 20]. But it is generated, by an instanton for $F = N-1$, and by gaugino condensation for other $F < N$. Going back to the $3-2$ model, an $SU(3)$ instanton generates the superpotential (43).

Finally, we get to the $SU(2)$ dynamics. In the limit $\Lambda_2 \gg \Lambda_3$, we have $SU(2)$ with two flavors. The classical moduli space of this theory is parametrized by the "mesons" $V_{ij} = Q_i \cdot Q_j$, $V_{i4} = Q_i \cdot L$. An $SU(2)$ instanton modifies this moduli space, so that, at the quantum level, the moduli space is given by the V's subject to the constraint

$$W = A\,(\epsilon^{i_1 i_2 i_3 i_4} V_{i_1 i_2} V_{i3 i_4} - \Lambda_2^4) = A(Z - \Lambda_2^4) \ . \tag{46}$$

where A is a Lagrange multiplier.

We can now use these different limits to obtain the full superpotential of the model, which is a function

$$W = W(X_A, Y, Z, \lambda, \Lambda_3^7, \Lambda_2^4) \ . \tag{47}$$

As in the $SU(N)$ example above, we want to use the global symmetry, which in this case is $U(1) \times U(1)_R$ to constrain this function. However λ, Λ_3 and Λ_2 are of course neutral under this symmetry, so that wouldn't work. Note that in our $SU(N)$ example this was not a problem, because there was only one parameter in the theory, Λ, and at the last step we could constrain the way Λ enters on

dimensional grounds. So we need symmetries under which λ, Λ_i are charged, i.e., global symmetries that are broken by the tree-level superpotential, and/or have global anomalies. In particular, we want to treat λ as a background field, or spurion, and use the fact that the superpotential cannot depend on $\lambda^\dagger$.

The simplest symmetries to consider the following: Introduce $U(1)_Q$ under which Q has charge 1, with all other fields neutral. Under this symmetry, λ has charge -1, Λ_3^7 has charge 2, and Λ_2^4 has charge 3. It is probably clear why λ has charge -1. We are introducing a "fake" symmetry and treating λ as a background field charged under this symmetry. For the superpotential to be invariant under $U(1)_Q$, λ must have charge -1. Let's now see why we can think of Λ_3^7 as having charge 2. The $U(1)_Q$ symmetry is anomalous. Therefore, if we rotate Q by this symmetry, we will shift the $SU(3)$ θ-angle. The shift is proportional to the number of $SU(3)$ fermion zero modes charged under the global symmetry. This number is 2, because Q also has an $SU(2)$ index. Finally, recall that

$$\Lambda^b = \mu^b e^{-\frac{8\pi^2}{g^2(\mu)}+i\theta} , \tag{48}$$

so under the anomalous rotation, Λ_3^7 has charge 2.

Exercise: Introduce similarly $U(1)_{\bar{Q}_1}$, $U(1)_{\bar{Q}_2}$, and $U(1)_L$, and compute the charges of λ, Λ_3^7, Λ_2^4 under these symmetries. Then use these symmetries, together with $U(1)_R$, to show that the superpotential is of the form

$$W = \frac{\Lambda_3^7}{Y} f(t_1, t_2) + A(Z - \Lambda_2^4)\, g(t_1, t_2) , \tag{49}$$

where f and g are general functions of

$$t_1 = \frac{\lambda X_2 Y}{\Lambda_3^7} , \quad t_2 = \frac{Z}{\Lambda_2^4} . \tag{50}$$

Now consider the limit

$$X_2 \, , \lambda_3 \, , \lambda \to 0 . \tag{51}$$

In this limit, t_1 and t_2 can take any value, and we know

$$W \to A(Z - \Lambda_2^4) . \tag{52}$$

Therefore $g(t_1, t_2) \equiv 1$. Now take

$$Y \to \infty , \quad \lambda \to 0 . \tag{53}$$

Again t_1 and t_2 can take any value. But for large Y VEVs, the gauge symmetry is completely Higgsed with the gauge bosons very heavy. The low-energy theory is therefore weakly coupled, and the superpotential is given by

$$W = \frac{\Lambda_3^7}{Y} + \lambda X_2 \tag{54}$$

so that $f(t_1, t_2) = 1 + t_1$. We then have the full superpotential

$$W = \frac{\Lambda_3^7}{Y} + A(Z - \Lambda_2^4) + \lambda X_2 . \tag{55}$$

Since

$$\frac{\partial W}{\partial X_2} \neq 0 \ , \tag{56}$$

supersymmetry is broken.

We assumed here that the Kähler potential is non-singular in X_2. This is justified because the theory is driven away from the origin by the first term of (54), so that the gauge symmetry is completely broken. We can then integrate out the heavy gauge bosons, and the low energy theory can be described in terms of the gauge invariants X_A, Y and Z. Note that, as a result, the tree-level superpotential becomes linear in the fields, just as in the O'Raifeartaigh model.

Finally, we note that we derived the non-renormalization theorem once again. The tree-level superpotential is not modified by perturbative corrections.

3.2.3 Calculable minimum

We established supersymmetry breaking by deriving the full superpotential of the theory. We can now choose parameters for which the minimum is calculable. For $\Lambda_3 \gg \Lambda_2$, $\lambda \ll 1$, Y gets a large VEV, and the gauge symmetry is completely broken. Because of the superpotential (54), Z gets mass and we can integrate it out[6], to get

$$W = \frac{\Lambda_3^7}{Y} + \lambda X_2 \ . \tag{57}$$

Since the theory is weakly coupled in this limit, the Kähler potential is just the canonical Kähler potential

$$Q^\dagger Q + \bar{Q}_A^\dagger \bar{Q}_A + L^\dagger L \ , \tag{58}$$

and we can calculate the potential, either in terms of the elementary fields or in terms of the classical gauge invariants X_A and Y (to use the latter, one needs to project (58) on the classical moduli space). In particular, it is easy to show that in terms of elementary fields, the typical VEV is $v \sim \lambda^{-1/7}\Lambda_3$ and $E_{vac} \sim \lambda^{5/14}\Lambda_3$. This demonstrates the general features of calculable minima mentioned at the beginning of this section. As we lower the superpotential coupling λ, the ground state is driven to large VEVs, for which the theory is weakly coupled. Note also that, as expected, the supersymmetry breaking scale is proportional to to the relevant strong coupling scale, (Λ_3 in this limit) and to some positive power of the Yukawa coupling λ.

We end this section with a few comments.

First, in this example, we were able to derive the exact superpotential of the theory and conclude from it that supersymmetry is broken. It would have been much easier to just consider the limit $\Lambda_3 \gg \Lambda_2$, $\lambda \ll 1$, and show that supersymmetry is broken as we did in section 3.2.3. In general, even if we

[6] Because Z vanishes classically, the term $Z^\dagger Z$ in the Kähler potential is suppressed by some power of Λ_2/v, where v is the typical VEV. Therefore the Z mass is enhanced by v/Λ_2, and we can indeed integrate it out.

can only establish supersymmetry breaking for some range of parameters, (say $\Lambda_3 \gg \Lambda_2$, $\lambda \ll 1$), we expect this to hold generally, because there should not be any phase transition as we vary the parameters of the theory. However, the details of the breaking, such as the supersymmetry-breaking scale, can be different.

Second, we used two examples to demonstrate the analysis of supersymmetry breaking. There is a long list of models that are known to break supersymmetry [5]. The analysis of these models involves many interesting ingredients and phenomena: quantum removal of flat directions, supersymmetry breaking without R symmetry, and the use of a Seiberg-dual theory to establish supersymmetry breaking, to name but a few. Unfortunately, there is no fundamental organizing principle that would allow us to systematically classify known models, or to guide us in the quest for new ones.

4 Mediating the breaking

We now know that supersymmetry can be broken, and that if broken dynamically, its scale is proportional to some strong coupling scale, Λ, which can be much lower than the Planck scale. In fact, this is all we need from the previous sections in order to discuss the mediation of supersymmetry breaking to the MSSM.

The MSSM contains many soft supersymmetry-breaking terms: scalar masses, gaugino masses, A-terms etc. This is often cited as a drawback of supersymmetry. But in any sensible theory, the soft terms must be generated by some underlying theory, and this underlying theory may have very few parameters. In fact, as we will see, if the soft terms are generated by anomaly-mediation, they are controlled by a *single* new parameter—the overall supersymmetry breaking scale.

The MSSM soft terms were discussed in detail in the lectures of Wagner, Masiero and Nir [21]. As we saw in these lectures, the soft terms determine the way we will observe supersymmetry in collider experiments, and are severely constrained by flavor changing processes. Here we will discuss several mechanisms for generating the soft terms

- Mediation by Planck-suppressed higher-dimension operators (a.k.a. "gravity mediation")
- Anomaly mediated supersymmetry breaking (AMSB)
- Gauge mediated supersymmetry breaking (GMSB)

We will focus on AMSB, because it is always present, and because it is probably the most tricky.

Suppose then that the fundamental theory contains, in addition to the MSSM, some fields and interactions that break supersymmetry (these are usually referred to as a supersymmetry breaking "sector", and the MSSM is sometimes referred to as the "visible sector"). We can think of the supersymmetry

breaking sector as the $3-2$ model, or the $SU(5)$ model we saw above, or even as a model with tree-level breaking, if we don't mind having very small parameters in the Lagrangian. The question is then: What do we need to do in order to communicate supersymmetry breaking to the MSSM, namely, generate the MSSM soft terms?

4.1 Mediating supersymmetry-breaking by Planck-suppressed operators

The short answer to this question is—nothing. The effective field theory below the Planck scale generically contains higher dimension operators that are generated when heavy states with masses of order the Planck scale are integrated out. These higher dimension terms couple the MSSM fields to the fields of the supersymmetry breaking sector. Denoting the MSSM matter superfields by Q_i, where i is a generation index, and a field of the supersymmetry breaking sector by X, the Kähler potential is then of the form

$$Q_i^\dagger Q_i + X^\dagger X + c_{ij}\frac{1}{M_{Pl}^2}X^\dagger X Q_i^\dagger Q_j + \cdots \ , \tag{59}$$

where c_{ij} are order-one coefficients. If X has a non-zero F-term, the last term of (59) gives rise to scalar masses for the Q's:

$$\left(m^2_{\tilde{Q}}\right)_{ij} = c_{ij}\left|\frac{F_x}{M_{Pl}}\right|^2 \ . \tag{60}$$

For the scalar masses to be around the electroweak scale we need

$$\frac{F_x}{M_{Pl}} \sim 100\mathrm{GeV} \ , \tag{61}$$

or $\sqrt{F_x} \sim 10^{11}$GeV. So it is very easy to generate the required scalar masses. However, there is no reason for the coefficients c_{ij} to be flavor blind. The fundamental theory above the Planck scale is certainly not flavor blind, because it must generate the fermion masses we observe. Generically then, this mechanism, which is usually referred to as "gravity mediation", leads to large flavor changing neutral currents. There are some solutions to this problem. One solution, which we heard about in Nir's lecture, uses flavor symmetries, with different generation fields transforming differently under the symmetry, leading to "alignment" of the fermion and sfermion mass matrices [22].

In fact, the name "gravity-mediation" is misleading, because the mass terms are not generated by purely gravitational interactions. Instead, they are mediated by heavy string states which couple to the MSSM and to supersymmetry-breaking fields with unknown couplings.

Can we suppress these dangerous contributions to the masses? One way to do this, is to suppress the coefficients c_{ij}. It is easy to do this if there are extra dimensions [23]. For example, if the MSSM is confined to a 3-brane, and the

supersymmetry breaking sector lives on a different 3-brane, separated by an extra dimension, then tree-level couplings of the the two sectors are exponentially suppressed, $c_{ij} \sim \exp(-MR)$, where R is the distance between the branes, and M is the mass of the heavy state that mediates the coupling. Such models are called sequestered models. [7]

Assume then that tree-level couplings of the MSSM and supersymmetry breaking sector are negligible. As it turns out, gravity automatically generates soft masses for the MSSM fields through the scale anomaly of the standard model. This time, the mediation of supersymmetry breaking is purely gravitational.

4.2 Anomaly mediated supersymmetry breaking

As we said above, we are assuming that apart from the MSSM, the theory contains a supersymmetry-breaking sector. Therefore, as mentioned in section 2.6, the scalar auxiliary field of the supergravity multiplet develops a non-zero VEV F_ϕ. The couplings of this auxiliary field to the MSSM are contained in eqn (32) which we repeat here for convenience

$$\mathcal{L} = \int d^2\theta \Phi^3 W(Q) + \int d^4\theta \Phi^\dagger \Phi K(Q^\dagger, e^V Q) + \int d^2\theta\, \tau\, W^\alpha W_\alpha \ . \tag{62}$$

Here Q denotes collectively the MSSM matter fields, and V stands for the MSSM gauge fields. Note that because

$$\Phi = 1 + F_\Phi \theta^2 \ , \tag{63}$$

F_ϕ has dimension one. We could instead write it as $F_\Phi = F/M_{Pl}$, where F is dimension-2 as usual.

At first sight, it seems that the non-zero F_Φ has no effect on the MSSM fields, because we can rotate it away by rescaling

$$Q \to \Phi^{-1} Q \ . \tag{64}$$

Note however that this assumes that the superpotential is trilinear in the fields, as is true for the MSSM apart from the μ term. If the superpotential contains a quadratic term then the rescaling gives, schematically,

$$\int d^2\theta \Phi^3 [Q^3 + MQ^2] \to \int d^2\theta [Q^3 + M\Phi Q^2] \ . \tag{65}$$

Thus, an explicit mass parameter would pick up one power of Φ

$$M \to M\Phi = M(1 + F_\Phi \theta^2) \ . \tag{66}$$

We will come back to this point often in the following. But as we said above, the MSSM classical Lagrangian is scale invariant—no mass parameter appears, and therefore the non-zero F_Φ has no effect.

[7] The c_{ij}'s can be suppressed in 4d theories too, using "conformal sequestering" [24].

This scale invariance is lost of course when we include quantum effects. The gauge and Yukawa couplings become scale dependent, and the dependence is controlled by the relevant β functions. We now have an explicit mass scale—the cut-off scale Λ_{UV}. As we saw above, this mass scale will pick up powers of Φ. Since the latter has a non-zero θ^2 component, we will obtain supersymmetry breaking masses for the MSSM fields [23, 25].

Consider first gaugino masses. These will come from

$$\int d^2\theta \frac{1}{4g^2(\frac{\mu}{\Lambda_{\rm UV}})} W^\alpha W_\alpha \tag{67}$$

since $\Lambda_{\rm UV}$ is rescaled by Φ (the simplest way to see this is to think of $\Lambda_{\rm UV}$ as the mass of regulator fields), (67) becomes

$$\int d^2\theta \frac{1}{4g^2(\frac{\mu}{\Lambda_{\rm UV}\Phi})} W^\alpha W_\alpha = \int d^2\theta \left[\frac{1}{4g^2_{\rm UV}} + \frac{b}{32\pi^2}\,\ln\frac{\mu}{\Lambda_{\rm UV}\Phi}\right] W^\alpha W_\alpha \ , \tag{68}$$

where b is the one-loop β function coefficient for the gauge coupling. Substituting (62) and expanding in θ, we get

$$\frac{1}{4g^2(\mu)} W^\alpha W_\alpha\Big|_{\theta^2} - \frac{b}{32\pi^2} F_\Phi \lambda^\alpha\lambda_\alpha \ . \tag{69}$$

The last term is a mass term for the gaugino. Going to canonical normalization for the gaugino,

$$m_\lambda(\mu) = \frac{b}{2\pi}\,\alpha(\mu) F_\Phi \ . \tag{70}$$

Exercise: scalar masses. Repeat this analysis for the scalars. Start from the Kähler potential

$$\int d^4\theta Z\left(\frac{\mu}{\Lambda_{\rm UV}}\right) Q^\dagger Q \ , \tag{71}$$

where Z is the wave-function renormalization. After the rescaling this becomes

$$\int d^4\theta Z\left(\frac{\mu}{\Lambda_{\rm UV}(\Phi^\dagger\Phi)^{1/2}}\right) Q^\dagger Q \ , \tag{72}$$

(The combination $(\Phi^\dagger\Phi)^{1/2}$ appears because Z is real). Expand this to obtain

$$\begin{aligned} m_0^2(\mu) &= -\frac{1}{4}\frac{\partial\gamma(\mu)}{\partial\ln\mu}\,|F_\Phi|^2 \\ &= \frac{1}{4}\left[\frac{b_g}{2\pi}\alpha_g^2\frac{\partial\gamma}{\partial\alpha_g} + \frac{b_\lambda}{2\pi}\alpha_\lambda^2\frac{\partial\gamma}{\partial\alpha_\lambda}\right]|F_\phi|^2 \end{aligned} \tag{73}$$

where

$$\gamma(\mu) = \frac{\partial\ln Z(\mu)}{\partial\ln(\mu)} \tag{74}$$

is the anomalous dimension, $\alpha_g = g^2/(4\pi)$, $\alpha_\lambda = \lambda^2/(4\pi)$, λ is a Yukawa coupling, and β_λ is its one-loop β function coefficient.

The AMSB masses (and A-terms, which can be derived similarly) are determined by the MSSM couplings, beta functions, and anomalous dimensions. The only new parameter that appears is F_Φ, which sets the overall scale. Since gaugino masses are generated at one-loop, and scalar masses squared are generated at two loops, the masses are comparable, of order a loop factor times F_Φ. Furthermore, the scalar masses are largely generation blind. Apart from third generation fields, for which flavor-changing constraints are rather weak, the masses are dominated by gauge contributions. Thus, FCNC's are not a problem. Finally, the expressions (70) and (73) are valid at any scale, and in particular, at low energies. Thus, AMSB is extremely elegant. Unfortunately, it predicts negative masses-squared for the sleptons, because $\beta_g < 0$ for $SU(2)$ and $SU(3)$.

So minimal AMSB does not work. Furthermore, we can already guess, from the fact that the soft terms can be calculated directly at low energies, that it will not be easy to modify them by introducing new physics at some high scale. We will explain this in detail in section 4.4. But before doing that, let's pause to consider gauge mediation. We will then use the results of this section together with the results of the next section to tackle the question of "fixing" anomaly mediation in section 4.4.

4.3 Gauge mediated supersymmetry breaking

In the last two sections, we assumed that the supersymmetry breaking sector and the MSSM only couple indirectly, either through higher-dimension operators, or through the supersymmetry breaking VEV of the supergravity multiplet. In this section, we will instead extend the MSSM, and couple it, mainly through gauge (but typically also through Yukawa) interactions, to the supersymmetry breaking sector. The main ingredient of gauge mediation are new fields, that are charged under the standard-model gauge group, and couple directly to the supersymmetry breaking sector, so that they get supersymmetry-breaking mass splittings at tree level. These fields are usually called the "messengers" of supersymmetry breaking. The MSSM scalars and gauginos obtain supersymmetry-breaking mass splittings at the loop level, from diagrams with messengers running in the loop.

We cannot go into detailed model building here. Instead, we will concentrate on the simplest set of messenger fields. Furthermore, to simplify the discussion, we will focus on the $SU(3)$ gauge interactions, and ignore $SU(2) \times U(1)$. Our discussion can be trivially extended to include these.

We then consider a "vector-like" pair of messengers, chiral superfields Q_3 and $\bar{Q}_3$, transforming as a 3 and $\bar{3}$ of $SU(3)$ respectively [26, 27]. The messengers couple to the supersymmetry-breaking sector through the superpotential

$$W_{mess} = X Q_3 \bar{Q}_3 \ , \tag{75}$$

where X is a standard-model singlet, with a non-zero VEV, $\langle X \rangle = M$ and F-term VEV, $\langle F_X \rangle = F \neq 0$. The Q_3, $\bar{Q}_3$ fermions then get mass M. The scalar

mass terms are of the form

$$M^2|\tilde{Q}_3|^2 + M^2|\tilde{\bar{Q}}_3|^2 + \left(F\tilde{Q}_3\tilde{\bar{Q}}_3 + h.c.\right) \ , \tag{76}$$

so that

$$m^2_{\tilde{Q}_3} = M^2 \pm F \ . \tag{77}$$

The gluinos then get mass at one loop (with the Q_3 scalar and fermion running in the loop)

$$m_\lambda = \frac{\alpha_3}{4\pi}\frac{F}{M} + \mathcal{O}\left(\frac{F}{M^2}\right)^2 \ , \tag{78}$$

and the squarks get masses at two loops,

$$m_0^2 \sim \left(\frac{\alpha_3}{4\pi}\right)^2 \left(\frac{F}{M}\right)^2 + \mathcal{O}\left(\frac{F}{M^2}\right)^4 \ , \tag{79}$$

We will see how to calculate these masses in the following.

The masses only depend on the SM gauge couplings, and are therefore flavor blind, so that there are no FCNC's. The gaugino and scalar masses are again comparable, and given by a loop factor times F/M. We therefore want F/M to be around $10^4 - 10^5$GeV. For M lower than, roughly, 10^{16}GeV, the M_{Pl} suppressed contributions we saw in section 4.1 are negligible. They would contribute soft masses of the order of F/M_{Pl}, at least two orders of magnitude below the gauge-mediated masses. (The AMBS masses are smaller by a loop factor.)

We will now see a nice trick [28] for calculating the GMSB soft masses, to leading order in the supersymmetry breaking, F/M^2. In the model we considered above, the masses are generated when the messengers are integrated out at $\langle X\rangle = M$. The effective theory for the gluinos below the messenger scale depends on M through the gauge coupling,

$$\mathcal{L} = \int d^2\theta \frac{1}{4g^2(\mu)} W^\alpha W_\alpha \ , \tag{80}$$

with

$$\frac{1}{g^2(\mu)} = \frac{b_H}{8\pi^2}\ln\frac{X}{\Lambda_{\rm UV}} + \frac{b_L}{8\pi^2}\ln\frac{\mu}{X} \ , \tag{81}$$

where b_H is the one-loop beta-function coefficient above M (MSSM $+Q_3+\bar{Q}_3$) and b_L is the one-loop beta function coefficient below M (MSSM). The key point is that we promoted the VEV of X to the field X. Since $X = M + \theta^2 F$ the situation is completely analogous to what we had in the previous section. We can Taylor expand in θ to get the gaugino mass

$$m_\lambda(\mu) = \frac{\alpha(\mu)}{4\pi}(b_L - b_H)\frac{F}{M} \ . \tag{82}$$

In our case, $b_L - b_H = 1$, so we recover (78). Similarly, starting with the quark kinetic term we can essentially repeat the derivation of scalar masses in AMSB, to get (79).

This concludes our short review of gauge mediation.

4.4 How NOT to fix AMSB

As we saw above, minimal AMSB gives rise to tachyonic sleptons. One might try to modify the slepton masses by adding some new physics at a high scale. We will now show that this has no effect on the masses at low scales. We assume here that the only source of supersymmetry breaking in the visible sector is anomaly mediation. For simplicity let us take the new fields to be the vector-like pair Q_3, $\bar{Q}_3$ of the previous section. We also add the superpotential

$$W = M Q_3 \bar{Q}_3 \ . \tag{83}$$

Now let us calculate the AMSB masses at low energies below M. For simplicity, we will consider gaugino masses only, but a similar discussion applies for scalar masses and A terms. Just above the scale M, the gaugino masses are given by the usual AMSB prediction (70)

$$m_\lambda(\mu) = \frac{b_H}{2\pi}\,\alpha(\mu) F_\Phi \qquad \text{for} \quad \mu > M \ , \tag{84}$$

where b_H is the beta function coefficient for the MSSM + Q_3, $\bar{Q}_3$,

$$b_H = b_{\text{MSSM}} + 1 \ .$$

At the scale M, we need to integrate out the heavy fields. But because the superpotential (83) contains an explicit mass parameter, these fields get supersymmetry-breaking mass splittings at tree-level

$$W = M Q_3 \bar{Q}_3 \to \Phi M Q_3 \bar{Q}_3 = (1 + F_\Phi \theta^2) M Q_3 \bar{Q}_3 \ , \tag{85}$$

with the fermions at M, and the scalars at $m^2 = M^2 \pm M F_\phi$. So Q_3 and $\bar{Q}_3$ behave just like the messengers of gauge mediation! We can calculate their contribution to the gaugino masses just as we did in the previous section. Clearly, the effect of this contribution is to precisely cancel the Q_3 $\bar{Q}_3$ part in b_H, so that below M, the gaugino mass is

$$m_\lambda(\mu) = \frac{b_{\text{MSSM}}}{2\pi}\,\alpha(\mu) F_\Phi \quad \text{for} \ \ \mu < M \ , \tag{86}$$

as in minimal AMSB. The heavy fields decouple completely and have no effect on the soft masses [23, 29, 30].

Note that it was crucial here that the new fields get mass in a supersymmetric manner. To emphasize this, let's give an even simpler argument for the decoupling. Consider the low-energy theory below M,

$$\int d^2\theta\, \tau(\mu, M, \Lambda_{\text{UV}})\, W^\alpha W_\alpha \ . \tag{87}$$

On dimensional grounds

$$\tau = \tau\left(\frac{\mu}{\Lambda_{\text{UV}}}, \frac{M}{\Lambda_{\text{UV}}}\right) \ . \tag{88}$$

Rescaling explicit mass scales by Φ

$$\tau\left(\frac{\mu}{\Lambda_{\rm UV}},\frac{M}{\Lambda_{\rm UV}}\right)\to\tau\left(\frac{\mu}{\Lambda_{\rm UV}\Phi},\frac{M\Phi}{\Lambda_{\rm UV}\Phi}\right)=\tau\left(\frac{\mu}{\Lambda_{\rm UV}\Phi},\frac{M}{\Lambda_{\rm UV}}\right)\ . \tag{89}$$

The Φ dependence cancels out completely in M so we recover the minimal AMSB prediction. Note that the cancellation only holds to leading order in F_Φ. The reason is that the AMSB masses are given fully by (70) and (73), with no corrections at higher order in F_Φ. In contrast, the "GMSB" contributions from integrating out Q_3 and $\bar{Q}_3$, do contain higher order corrections, that are not captured by the trick we saw in the previous section.

The same discussion applies to different heavy thresholds, and in particular those associated with D terms, which have attracted some attention lately [31]. The basic idea [30] is to get slepton masses by adding a new $U(1)$ symmetry, under which the MSSM matter fields are charged. Probably the simplest model [30] involves new fields $h_\pm$, $\xi_\pm$, with charges ± 1 under the $U(1)$, as well as gauge singlets n_i, $i=1,2$, and S, with the superpotential

$$W=S(\lambda h_+h_- - M^2)+y_1n_1h_+\xi_- + y_2n_2h_-\xi_+\ . \tag{90}$$

Because of the first term, h_+ and h_- obtain VEVs and break the $U(1)$. All new fields get mass either by the Higgs mechanism or through the superpotential. With no supersymmetry breaking, h_+ and h_- get equal VEVs. However, assuming that there is some supersymmetry breaking sector, all fields get supersymmetry breaking masses through AMSB. In particular, for $y_1\neq y_2$, h_1 and h_2 have different soft masses and therefore different VEVs, so that the $U(1)$ D-term is non-zero. If the sleptons are charged under the $U(1)$, one might naively think that the D term affects the slepton masses. But as explained in [30], this is not the case. The model described above has no effect on the soft masses at low energy, to leading order in the supersymmetry breaking, F/M^2. In [30], the surviving F^4/M^2 contributions were used in order to generate acceptable slepton masses, using the fact that these enter scalar masses-squared at *one*-loop. The scale M was generated dynamically from F, so that it was roughly two orders of magnitude (an inverse loop-factor) above F.

To conclude, we see that we cannot modify the AMSB predictions at leading order in F_Φ using new heavy thresholds that get mass in the limit of unbroken supersymmetry. Clearly then, there are two possible approaches to fixing AMSB models: One is to use higher order terms in the supersymmetry breaking F_Φ. The second is to introduce thresholds for which some fields remain light in the limit of unbroken supersymmetry.

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